

1. Daniel decides he desires something deciduous, deciding on double dogwood trees in his yard. The trees are 80 feet apart, one tree is 40 feet tall, and the other is 20 feet tall. Assuming his yard is perfectly flat, how many feet are there between the tops of the trees?

We can imagine a right triangle with base of 80 and height of $40 - 20 = 20$. We can then use the Pythagorean Theorem on this triangle to find the distance as $\sqrt{80^2 + 20^2} = \sqrt{6400 + 400} = \sqrt{6800} = 20\sqrt{17}$. The distance between the tops of the trees is $\boxed{C, 20\sqrt{17}}$.

2. The American Bladdernut is a deciduous shrub that is very real. Shawn decides to get his hands on a bladdernut fruit. If the fruit is shaped like a perfect sphere with radius 6, what is the volume of Shawn's bladdernut?

We use the formula to find the volume of a sphere, which is $\frac{4}{3}\pi r^3$. If we plug in the radius of 6, then we get $\frac{4}{3}\pi \cdot 6^3 = \frac{4}{3}\pi \cdot 216 = \frac{4 \cdot 216}{3}\pi = 4 \cdot 72\pi = 288\pi$. The volume of the sphere is $\boxed{B, 288\pi}$.

3. Canadians love their maple trees! Allen wants to find the area of his maple leaf. The maple leaf is an equilateral triangle with side length 4, then it sprouts two equilateral triangles of half its length, then each of those sprouts two 30-60-90 right triangles, with the side opposite the 60° angle having length equal to half of the smaller equilateral triangles' side lengths. The stem is a rectangle of length 2 and width $\frac{1}{2}$. What is the area of Allen's maple leaf?

Start by calculating the area of the large equilateral triangle, by the equilateral triangle area formula, $(\frac{\sqrt{3}}{4}s^2)$, we have that the area is $4\sqrt{3}$. The two smaller triangles have half the lengths of the large equilateral triangle, so each area is scaled down by a factor of 4, leaving the areas of both of them as $2\sqrt{3}$. Then, observe that each small right triangle has $\frac{1}{6}$ the area of the smaller equilateral triangles. This leaves the sum of the 6 of them having area $\sqrt{3}$. Then, add in the rectangle's area to get $\boxed{C, 7\sqrt{3} + 1}$.

4. Johnny was preparing for his run through the forest, so he prepares fat stacks of Johnnycakes. His Johnnycakes are cylinders with height 1, and he makes one Johnnycake with a radius of 5, 4, 3, 2, and 1. He then stacks them on top of each other concentrically, starting with the radius 5 at the bottom, then the 4, etc. What is the surface area of his stack of Johnnycakes? Include the bottom.

Viewing from the top we get 16π in surface area, similarly with the bottom, and viewing from the sides gets us 20π , for a grand total of $\boxed{B, 52\pi}$.

5. A square tree has leaves in the shape of squares that have integer area. If the leaves can be placed on the coordinate plane such that all of its square vertices lie on lattice points, which one of the answer choices could be the area of the leaves? A lattice point is a point where both of its coordinates are integers.

A square with area n where n is an integer has side lengths \sqrt{n} . By the distance formula and the lattice condition, we must find the answer choice that is the sum of two squares. This is $\boxed{D, 73}$, which is representable as $64 + 9$.

6. You are walking to Dr. Deciduous's house when suddenly, Karen blocks your path and asks you to find the area of a triangle with side lengths 13, 14, and 15. What should you tell her?

We can use Heron's formula on the triangle. The semiperimeter is 21, so the area of the triangle is

$$\sqrt{21(21-13)(21-14)(21-15)} = \boxed{D, 84}.$$

7. Professor Oak Tree is making a deciduous tree-shaped cake for research purposes, and he uses frosting to make the leaves. He pipes his frosting onto the cake using a cake piper. His cake piper is a cone with a height of 60, and a radius of 15. What is the volume of Professor Oak Tree's cake piper?

The cone area formula is $\frac{1}{3}$ of the cylinder volume formula, using this we get that the volume is $\boxed{D, 4500\pi}$.

8. Chi is doing a watercolor drawing of a beech tree, except she messes up and spills her colors onto the paper. The end result is the watercolor in the shape of a circle of radius 6, and a square of side length $6\sqrt{3}$ overlapping concentrically. What is the area of the spill?

The diameter of the circle is equal to the diagonal of the square, which is $6\sqrt{2}$. Then the area of the circle is $(3\sqrt{2})^2\pi = 18\pi$, and since the area of the square is $6^2 = 36$, the answer is $\boxed{E, 18\pi - 36}$.

9. Many scientists are wondering how to combat climate change, but Professor Oak Tree thinks he has the perfect solution! Oak Tree knows he needs to combat carbon emissions by developing carbon sinks, like oak trees! Professor Oak Tree makes an oak tree farm in the shape of a square formed by connecting the midpoints of every other side of a regular octagon. If the side length of the octagon is $4\sqrt{2}$, what is the area of the farm?

Extend the sides of the octagon that contain the vertices of the square to form a larger square. The side length of this square is $4\sqrt{2} + 4 + 4 = 8 + 4\sqrt{2}$. From symmetry, the vertices of the initial square are the midpoints of the sides of this larger square. Then the answer is $\frac{1}{2}(8 + 4\sqrt{2})^2 = \boxed{D, 48 + 32\sqrt{2}}$.

10. Dr. Deciduous needs some medicine, but he needs to go to the his office to get a prescription first. His office is anywhere on the x -axis, Dr. Deciduous is at $A = (0, 1)$, and the pharmacy is at $B = (3, 3)$. Dr. Deciduous, being excited to get his medicine, figures out the fastest way to get to the his office before getting to the pharmacy. What is the length of AC , where C is the location of his office?

Note that if we reflect the pharmacy over the x -axis and draw a straight line between Dr. Deciduous and the pharmacy, by reflecting back we get the shortest path possible. Reflecting B over the x -axis to $B' = (3, -3)$, we want to find the portion of $\overline{AB'}$ above the x -axis. From the distance formula, $AB' = 5$, so from similar triangles, $AC = \frac{5 \cdot 1}{1+3} = \boxed{A, \frac{5}{4}}$.

11. Emre loves geometry, deciduous forests, and cake! Inspired by Professor Oak Tree, he also made a cake in the shape of a tree trunk. If the cake is a cylinder with radius 3 and the height 6, what is the volume of the cake?

The radius of the cylinder is 3 and the height 6, leaving the volume of the cylinder equal to $\boxed{E, 54\pi}$.

12. Shawn walks from his house at $(8, 0)$, stops in the forest at $(16, 6)$ for 4 hours, naps at a river at $(16, -6)$ for 3 hours, and goes back home. Assume that Shawn goes from his house to the forest using the shortest path possible. He also uses the shortest path possible to go from the forest to the river to his home. If Shawn walks at 1 unit per hour, how many hours is he away from home for?

The distance between Shawn's house and the forest is 10 by the distance formula, the distance between the forest and the river is 12, and the distance between the river and his house is 10. Add in the hours he stayed, this is $7 + 10 + 10 + 12 = \boxed{D, 39}$.

13. Modern philosopher Daniel Slang, a Tree Sage and a teacher of wisdom, gave this puzzle to his students. Let $\triangle ABC$ be an isosceles triangle with $AB = AC$ and $\angle BAC = 50^\circ$. If the angle bisectors of $\angle ABC$ and $\angle BAC$ intersect at I , what is $\angle BIC$?

Note that

$$\angle BIC = 180^\circ - (\angle IBC + \angle ICB) = 180^\circ - \frac{1}{2}(\angle ABC + \angle ACB) = 180^\circ - \frac{1}{2}(180^\circ - \angle BAC) = 90^\circ + \frac{1}{2}\angle BAC.$$

Then $\angle BIC = 90^\circ + 25^\circ = \boxed{D, 115^\circ}$, as desired.

14. Canada has some deciduous forests, but it also has a lot of snow. Allen walks into a forest and sees a snowflake growing bigger! Originally an equilateral triangle of side length 4, the snowflake adds an equilateral triangle in the middle of each side, and then repeats this indefinitely. What area will the snowflake approach? The first two growths are shown.

The large equilateral triangle has area $4\sqrt{3}$, then the area of the 3 attached onto it combined have area $\frac{4}{3}\sqrt{3}$. Note that every iteration forward each added triangle has $\frac{1}{9}$ the areas of the previous, but there's 2 times more, leaving us with a geometric series of ratio $\frac{2}{9}$. Each triangle of the second iteration has area $\frac{4}{81}\sqrt{3}$, and there's 6 of them, so for this iteration the triangles have area $\frac{8}{27}$. By the geometric series formula, the sum $\sum_{i=0}^{\infty} (\frac{2}{9})^i = \frac{9}{7}$. So, the area of all iterations after the first iteration is $\frac{8\sqrt{3}}{21}$, adding in the original triangle and the first iteration leaves the answer as $\boxed{C, \frac{40\sqrt{3}}{7}}$.

15. Shawn is similar to a fractal, in that when you look closely at his muscles, even his muscles have muscles. If his first muscle has volume π , his second muscle has volume $\frac{\pi}{3}$, and so on with each muscle having $\frac{1}{3}$ rd the volume of the previous one, what is the volume of Shawn's muscles?

This is an infinite geometric series with first term π and common ratio $\frac{1}{3}$, so the answer is $\frac{\pi}{1-\frac{1}{3}} = \boxed{B, \frac{3\pi}{2}}$.

16. Dr. Deciduous finds a tree at the point $(-10, -20)$. Upon closer inspection, Dr. Deciduous makes a horrid observation: the tree is actually evergreen! In a fit of rage, Dr. Deciduous throws the tree around, in such a way that can be represented as reflecting over line $y = x$, then translating it up by 4, and finally rotating it 90° counterclockwise about the origin. Where is the tree located now?

Reflecting over $y = x$ swap the coordinates of a point, so $(-10, -20)$ goes to $(-20, -10)$. Translating up by 4 gives $(-20, -6)$. Finally, rotating 90° counterclockwise moves the point from the third to the fourth quadrant, which we can see from graphing to be $\boxed{A, (6, -20)}$.

17. Ziad was playing his favorite mobile game, Deciduous Royale. He throws a fireball at the opposing side's deciduous treehouse, destroying it. If the fireball leaves a hemispherical crater with radius 3, what is the volume of space inside the crater?

The volume of a sphere with radius r is $\frac{4\pi}{3}r^3$, so the volume of the hemisphere with radius 3 is $\frac{2\pi}{3} \cdot 3^3 = \boxed{B, 18\pi}$.

18. Daniel goes to Costco, Walmart, Publix, and Kohl's to get supplies to grow his deciduous double dogwood trees. Costco, Walmart, Publix, and Kohl's form a quadrilateral with vertices $(-15, 0)$, $(-1, -8)$, $(6, -7)$, $(3, 4)$. Daniel drives from his house to Costco, back home, to Walmart, back home, and so on until he arrives home after the last store. Where is his house if he drives the least possible amount? Assume he drives in straight lines.

The problem is equivalent to finding the point which has the least total distance between itself and the other vertices. We have a convex quadrilateral, draw in the diagonals. The shortest distance between two points is a straight line, if we place Daniel's house at the intersection of the diagonals then the sum of the distances to the vertices is the sum of the lengths of the diagonals. Anywhere else and it will not be optimal, as the sums of the distances between our point and opposite vertices on the quadrilateral will not be optimal. So, it is at the intersection, calculating that here we get the answer to be $\boxed{C, (0, -5)}$.

19. Scientists have found a "squirecle" shaped apple. A squirecle is defined by a radius and a length. Divide a circle of the radius chosen into fourths, then add rectangles with sides equal to the radius and length chosen between the fourths as shown in the diagram. Finally, add a square in the middle with the side length equal to the length. What is the area of a squirecle with length 8 and radius 2?

The area is the sum of the square in the middle with the circle's area and the four rectangles. Summing, we have $64 + 4\pi + 64$. Therefore, the answer is $\boxed{B, 128 + 4\pi}$.

20. Shayan and Daniel are playing basketball on their deciduous court, and Daniel blocks Shayan's shot with his entire hand. Shayan gets mad, and he punches Daniel's square head so hard it rotates 90° about its center. If Daniel's head has an area of 100, what is the area of the region Daniel's head traces as it rotates (include Daniel's head as part of the region traced)? Assume that Daniel is two-dimensional.

This rotation will map every vertex to its adjacent one, so the square will trace out its whole circumcircle. Since the square has area 100, its side length is 10 and diagonal is $10\sqrt{2}$, which is also the diameter of its circumcircle. Then its radius is $5\sqrt{2}$, and the answer is $(5\sqrt{2})^2\pi = \boxed{B, 50\pi}$.

21. Shawn has several curves, one of which being the curve $y = \sqrt{-x^2 + 4}$. Shawn wants to figure out the area between his curve and the x -axis from -2 to 2 , but Shawn doesn't know how. What is the closest integer to the area between Shawn's curve and the x -axis from -2 to 2 ?

Squaring both sides of the equation and rearranging gives $x^2 + y^2 = 4$, which is a circle with radius 2. However, y is a square root and cannot be negative, so the desired area is actually a semicircle. Then the area is $\frac{1}{2} \cdot 2^2 \cdot \pi = 2\pi \approx 6.28$, so the answer is $\boxed{C, 6}$.

22. Darth Deciduous is up to no good! To thwart his evil plans, you must locate his office located at the orthocenter of his giant triangular house! Darth Deciduous's house has vertices at $(-18, 0)$, $(0, 36)$, and $(24, 0)$. Where is his office located?

The line perpendicular to the side with vertices $(-18, 0)$ and $(24, 0)$ that passes through $(0, 36)$ is the y -axis. Find where another one of the altitudes intersects the y -axis gives $\boxed{D, (0, 12)}$.

23. Professor Oak Tree planted 3 oak trees A , B , C , where $AB = AC = 5$ and $BC = 8$. If G is the centroid of $\triangle ABC$, what is $\cos \angle BGC$?

Let M denote the midpoint of \overline{BC} . Since $AB = AC$, $\triangle ABM$ is right, so $AM = \sqrt{5^2 - 4^2} = 3$. Since G is the centroid of $\triangle ABC$, we have $AG : GM = 2 : 1$, so $AM = 1$. This gives $GB = GC = \sqrt{4^2 + 1^2} = \sqrt{17}$. Finally, from Law of Cosines on $\triangle GBC$, we have

$$\cos \angle BGC = \frac{(\sqrt{17})^2 + (\sqrt{17})^2 - 8^2}{2(\sqrt{17})(\sqrt{17})} = -\frac{15}{17},$$

and the answer is $\boxed{B, -\frac{15}{17}}$.

24. Jonathan decides enough is enough, and he lights his local deciduous forest on fire. The fire radiated outward from where Jonathan lit his first tree in every direction for 20 seconds, which was enough to burn down the entire forest. If the fire radiated outward at a speed of 1 unit per second, what is the maximum possible area of the forest?

It cannot spread faster than 1 unit a second, so the fire cannot reach anywhere farther than 20 units away from Johnny. Therefore, the entire forest is contained inside a circle with radius 20, so the maximum possible area is $20^2 \cdot \pi = \boxed{A, 400\pi}$.

25. Roger finds a peculiar tree in the shape of a hexagon! What is the sum of the angles in a hexagon in degrees?

A hexagon has 6 sides, so the sum of its angles in degrees is $180(6 - 2) = \boxed{D, 720}$.

26. Dr. Deciduous got angry at Jonathan for burning down his sacred deciduous forest, deciding to take revenge. Dr. Deciduous decided that if Jonathan can solve his research problem for him, then he will let Johnny go: Find the locus of points in the coordinate plane that satisfies the equation $x^3 + y^3 + x^2y + xy^2 - x - y = 0$.

Factoring, we get this is equal to $(x^2 + y^2 - 1)(x + y) = 0$. This means either $x^2 + y^2 = 1$, or $y = -x$. This is the union of a line and a circle, the answer is $\boxed{A, \text{A circle and a line}}$.

27. Dr. Deciduous finds an arrangement of trees that forms the shape of three regular polygons that each share a side with the other two polygons and all share a common vertex. If two of the polygons are a square and a hexagon, how many sides does the third polygon have?

Suppose the answer is n . Then

$$150 = 180 - \frac{360}{n} \implies \frac{360}{n} = 30 \implies n = \frac{360}{30} = \boxed{C, 12}.$$

28. Sophie and Daniel are sitting in a tree when Sophie pushes Daniel off. The tree can be represented by a cylinder with equal diameter and height, topped with a sphere with radius equal to the the cylinder's diameter. If the tree is 36 feet tall, what is the volume of the tree in ft^3 ?

The height of the structure is the diameter of the sphere added with the height of the cylinder, since the diameter of the sphere is double the double of the cylinder, the diameter of the cylinder is 12, and so the radius is 6. Calculating the cylinder's volume yields 432π , and the volume of the sphere is 2304π , totaling to $\boxed{D, 2736\pi}$.

29. Math is one of Jonathan's favorite subjects, rivaled only by environmental science. Being an environmental science enthusiast, Jonathan loved and wished to protect the deciduous forests of the world, but now he is mad that he didn't get into his favorite class! So, Johnny goes to the local forest, chops down some trees, and places the logs in his plastic box. If the logs are $18 \times 4 \times 4$ rectangular prisms and the box is an $18 \times 10 \times 8$ rectangular prism, what is the most amount of logs that can fit in the box?

The only way logs will fit completely in the box is if they are arranged so the length of the logs fit into the length of the box. Then we just want to find the number of 4×4 squares that can fit in a 10×8 rectangle. The best way to do this is to have a 2×2 arrangement of 4 squares, forming a larger 8×8 square that clearly fits. Then the answer is $C, 4$.

30. The answer is not not not not A, not B, not not not C, not not not not not not not D, and not not not not not E. What is the answer?

With negations, an odd number of negations means the statement is not correct, and an even number of negations means the statement is correct. A has 4 negations, B has 1, C has 3, D has 7, and E has 5. Since A is the only one with an even number of negations, the answer is $A, \text{Pick me!}$.